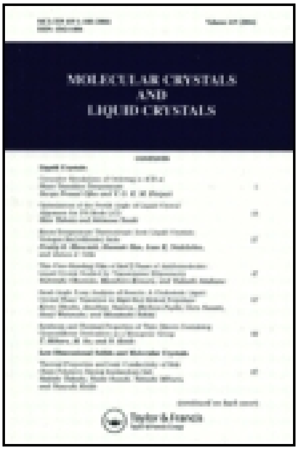

Molecular Crystals and Liquid Crystals Incorporating Nonlinear Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl17

Theretical Considerations on the Determinstion of the Tilt Angle of a Nematic Liquid Crystal at the Surface of a Substrate by Polarization Azimuth Measurement

Klaus Eidener [a] , Gudrun Mayer [a] & Rainer Schuster [a]

[a] Karl-Marx-Universität leipzig Sektion physik, 7010, leipzig, linnéstr. 5, G. D. R.

Version of record first published: 13 Dec 2006.

Mol. Cryst. Liq. Cryst., 1987, Vol. 152 pp. 447-452
Photocopying permitted by license only

Printed in the United States of America

THEORETICAL CONSIDERATIONS ON THE DETERMINATION OF THE TILT ANGLE OF A NEMATIC LIQUID CRYSTAL AT THE SURFACE OF A SUBSTRATE BY POLARIZATION AZIMUTH MEASUREMENT

KLAUS EIDNER, GUDRUN MAYER, and RAINER SCHUSTER
Karl-Marx-Universität Leipzig, Sektion Physik, 7010 Leipzig, Linnéstr. 5, G. D. R.

Abstract The behaviour of a plane monochromatic wave is described in the case of reflection and refraction at a boundary plane between an anisotropic and an isotropic medium. For a nematic liquid crystal the dependence of the azimuth of polarization of a transmitted wave in glass on the liquid crystal director orientation at the glass surface is derived.

INTRODUCTION

We have presented[1] a new measuring method for a direct determination of the tilt angle of a deformed nematic liquid crystal (LC) at a substrate surface by measuring the azimuth of polarization of a transmitted wave. This method is based on well-known laws of light propagation in anisotropic media. However, their application to special cases sometimes demands expansive derivations. In view of interesting applications to the determination of LC-parameters we want to treat the problem of refraction at a boundary plane between an anisotropic and an isotropic medium more detailed and derive the dependence of the azimuth of polarization of a transmitted wave on the orientation of the local opic axis (the director) at the boundary plane (the substrate surface).

THEORY

We consider a plane monochromatic wave

$$\underline{A} = A_M \underline{j_A} \exp(i\varphi) \qquad (1)$$

propagating in an anisotropic medium. Here $\underline{A}$ are the vectors of the electric ($\underline{E}$) and magnetic fields ($\underline{H}$), A_M are their amplitudes, $\underline{j_A}$ are the unit vectors of the directions of vibration, $\varphi = \omega t - \underline{Kr}$ is the phase and $\underline{K} = (\omega/c)\underline{k} = (\omega/c)n\underline{e}$ is the wave vector with the index of refraction n and the unit wave normal $\underline{e}$. This wave impinges on a boundary plane (xy-plane) between anisotropic and isotropic media. It gives rise to two reflected waves because of the double refraction in the crystal and one transmitted wave in the isotropic medium.

Now we take into account the boundary conditions of Maxwell's theory, which demand the continuity of the tangential components of the field vectors (x,y-components). We write them in the form

$$A_{\alpha,1} + A_{\alpha,2} + A_{\alpha,3} - A_{\alpha,4} = 0 \quad ; \quad \alpha = x, y. \qquad (2)$$

(Index 1 means incident wave, index 2 extraordinary reflected wave, index 3 ordinary reflected wave and index 4 transmitted wave.)

By replacing the components of the magnetic vector H_x and H_y using Maxwell's equation $\underline{H} = (\underline{k}\times\underline{E})/(\mu_0 c)$, we can write Eq. (2) in the form of a homogeneous linear system of equations

$$M \cdot V = 0 \tag{3}$$

with

$$M = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ 1 & 1 & 1 & -1 \end{pmatrix}, \tag{4}$$

$$\begin{aligned} a_m &= k_{ym} E_{zm}/E_{ym} - k_{zm} \\ b_m &= k_{zm} E_{xm}/E_{ym} - k_{xm} E_{zm}/E_{ym} \\ c_m &= E_{xm}/E_{ym} \quad , \quad m = \overline{1,4} \end{aligned} \tag{5}$$

and $V = (E_{y1}, E_{y2}, E_{y3}, E_{y4})^T$. From the condition

$$\det M = 0 \tag{6}$$

we get the connection between the wave properties and the properties of the media.

For the determination of the elements of the matrix M (Eq. (5)) we use:

1. the continuity of the tangential components of the wave vector at the boundary,
2. the reflection law for the extraordinary wave

$$\mathrm{ctg}\,\varphi_1 - \mathrm{ctg}\,\varphi_2 = 2\varepsilon_{xz}/\varepsilon_{zz} \tag{7}$$

(where φ_1 and φ_2 are the angles between wave normal and z-axis of the incident wave and the extraordinary reflected wave, respectively, ε_{xz} and ε_{zz} are components of the dielectric tensor $\underline{\underline{\varepsilon}}$),

3. the expression for the transmitted wave in the medium

$$\underline{E}_4 = E_{M4}\,(-k_{z4}\cos\chi/n_i,\ \sin\chi,\ k_{x4}\cos\chi/n_i), \tag{8}$$

with χ being the angle between the plane of vibration of the electric vector in the isotropic medium and the plane of incidence (xz-plane), and n_i the index of refraction of the isotropic medium, and

4. the relations for the unit vectors of the directions of vibration $\underline{j}_A$.

In the case of the ordinary ray we have

$$\underline{j}_{E_0} = N_{E_0}\,(\underline{k}\times\underline{L}) \tag{9}$$

and in the case of the extraordinary ray

$$\underline{j}_{E_e} = N_{E_e}\cdot\underline{\underline{\varepsilon}}^{-1}\cdot(\underline{j}_{E_0}\times\underline{k}). \tag{10}$$

Here $\underline{L}$ is the direction of the optic axis, N_{E_0} and N_{E_e} are coefficients of normalization. Decomposing $\underline{k}$ in components perpendicular ($\underline{k}_\perp$) and parallel to the optic axis,

$$\underline{k} = \underline{k}_\perp + (\underline{k}\underline{L})\underline{L}, \tag{11}$$

we obtain after some transformations

$$\underline{j}_{Ee} = N_{Ee}\,((1/n^2)\underline{L}-(1/n_o^2)(\underline{e}\underline{L})\underline{e})$$
$$\overline{1/n^2} = (1/n_e^2)\sin^2\gamma+(1/n_o^2)\cos^2\gamma \qquad (12)$$
$$\cos\gamma = \underline{e}\underline{L}$$

where n_o and n_e are the ordinary and the extraordinary indices of refraction in the anisotropic medium, respectively.

Now we regard a nematic LC under the following conditions:

1. The LC is deformed in the yz-plane by an external magnetic field. The deformation is described by the z-dependent angle $\Theta(z)$ of the director with respect to the z-axis. We have $\underline{L} = (0, \sin\Theta, \cos\Theta)$.
2. In this medium an extraordinary wave propagates in the xz-plane perpendicular to the plane of deformation with a wave vector $\underline{k} = (k_x, 0, k_z)$.
3. The deformation is sufficiently small so that a wave originally polarized in the principal plane remains linearly polarized. Its plane of vibration rotates with rotating principal plane (Maugiun limit).

Eq. (6) we can write

$$\begin{vmatrix} k_{z1} & -k_{z1} & -k_{z3} & k_{z4} \\ -k_x ct\Theta_s & -k_x ct\Theta_s & \dfrac{n_o^2\tan\Theta_s}{k_x} & n_i ct\chi \\ \dfrac{k_x k_{z1} ct\Theta_s}{n_o^2} & \dfrac{-k_x k_{z1} ct\Theta_s}{n_o^2} & \dfrac{k_{z3}\tan\Theta_s}{k_x} & \dfrac{k_{z4} ct\chi}{n_i} \\ 1 & 1 & 1 & -1 \end{vmatrix} = 0 \qquad (13)$$

where Θ_s is the tilt angle of the director at the substrate surface. Eq. (13) becomes

$$\tan\chi = \tan\Theta_s\,(k_x^2 - k_{24}k_{23})/(n_i k_x), \qquad (14)$$

with $k_{24}^2 = n_i^2 - k_x^2$, $k_{23}^2 = n_o^2 - k_x^2$ and $k_x = n_i \sin\alpha$. α is the refraction angle in the isotropic medium.

Using relation (14) we are able to determine directly the tilt angle at the surface Θ_s by measuring the azimuth of polarization of the transmitted wave in the isotropic medium χ. Our method of tilt angle determination[1] is based on the application of Eq. (14) in the special case of single internal reflection[2] of the extraordinary wave in the anisotropic medium.

ACKNOWLEDGEMENT

The authors are grateful to Prof. A. Lösche and Doz. Dr. H. Schmiedel for useful discussions.

REFERENCES

1. K. Eidner, G. Mayer, and R. Schuster, Mol. Cryst. Liq. Cryst., to be published.
2. F. Pockels, Lehrbuch der Kristalloptik (B.G. Teubner, Leipzig, 1906), p. 96.